

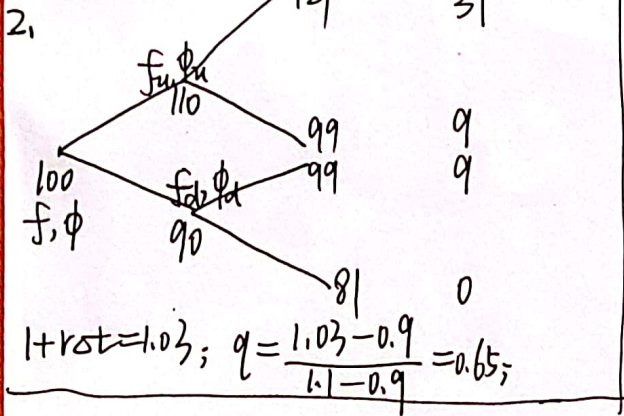
$$E(e^x) = \int_{-\infty}^{\infty} e^x \cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} e^{\frac{(x-\mu)^2 - 2\sigma^2 x}{-2\sigma^2}} dx$$

$$(x-\mu)^2 - 2\sigma^2 x = x^2 + \mu^2 - 2\mu x - 2\sigma^2 x = (x - (\mu + \sigma^2))^2 - (\mu + \sigma^2)^2 + \mu^2 = (x - (\mu + \sigma^2))^2 - \sigma^2 - 2\mu\sigma^2$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} e^{-\frac{(x - (\mu + \sigma^2))^2}{2\sigma^2}} \cdot e^{\frac{\sigma^2 + 2\mu\sigma^2}{2\sigma^2}} dx$$

$$= e^{\frac{\sigma^2}{2} + \mu} \cdot \text{density function}$$



$$f_u = \max(20, 1.03^{-1}(0.65 \times 31 + 0.35 \times 9)) = 22.62$$

$(110 - 90)^+$

$$\phi_u = \frac{31 - 9}{121 - 99} = 1$$

$$f_d = \max(0, 1.03^{-1}(0.65 \times 9 + 0.35 \times 0)) = 5.68$$

$(90 - 90)^+$

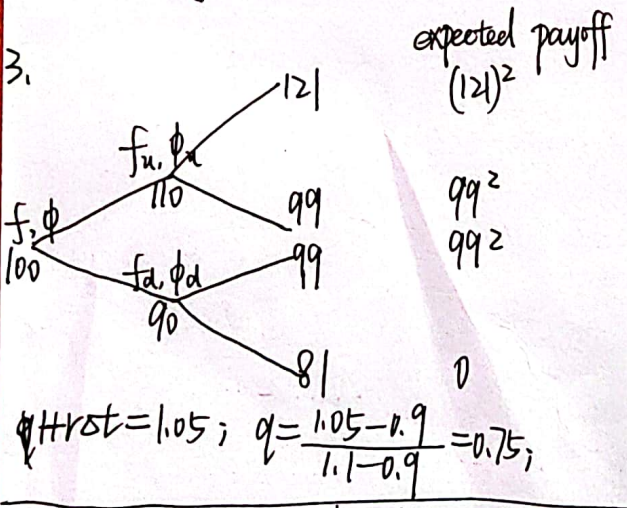
$$\phi_d = \frac{9 - 0}{99 - 81} = \frac{1}{2}$$

$$f = \max(10, 1.03^{-1}(0.65 \times 22.62 + 0.35 \times 5.68)) = 16.20$$

$$\phi = \frac{22.62 - 5.68}{110 - 90} = 0.85$$

Therefore, price is 16.2 at time t_0 .

Strategy: At time t_0 , you should borrow 68.8 from bank 0.85 share of stock. If the price goes up at time t_1 , you should borrow 16.5 from bank and buy extra 0.15 share of stock. Otherwise, you should sell $0.85 - 0.5 = 0.35$ share of stock and deposit all money into bank.



$$f_u = 1.05^{-1}(0.75 \times 121^2 + 0.25 \times 99^2) = 12791.43$$

$$\phi_u = \frac{121^2 - 99^2}{121 - 99} = 220$$

$$f_d = 1.05^{-1}(0.75 \times 99^2 + 0.25 \times 0) = 7000.71$$

$$\phi_d = \frac{99^2 - 0}{99 - 81} = 544.5$$

$$f = 1.05^{-1}(0.75 \times 12791.43 + 0.25 \times 7000.71) = 10803.57$$

$$\phi = \frac{12791.43 - 7000.71}{110 - 90} = 289.54$$

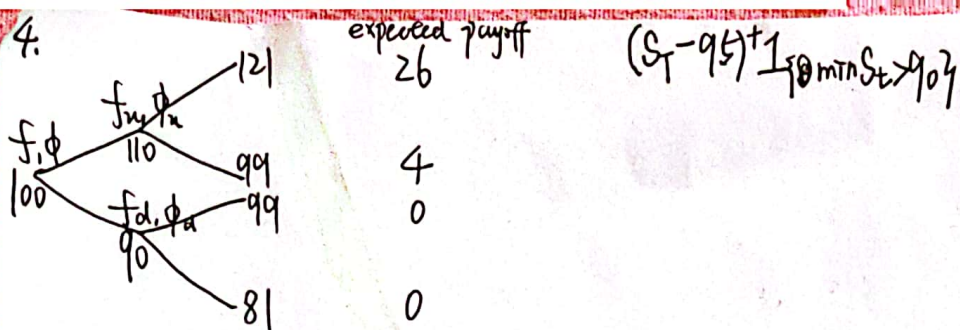
Therefore, the price should be 10803.57.

Strategy: t_0 : borrow 18150.43 from bank and buy 289.54 share of stock.

t_1 : \rightarrow sell 69.54 share of stock and deposit all cash.

\rightarrow borrow 22946.4 from bank and buy 254.96 share of stock.





$$H_{\text{rot}} = 1.05; \quad q = \frac{1.05 - 0.9}{1.1 - 0.9} = 0.75;$$

$$f_u = 1.05^{-1} (0.75 \times 26 + 0.25 \times 4) = 19.52$$

$$\phi_u = \frac{26 - 4}{121 - 99} = 1$$

$$f_d = 1.05^{-1} (0.75 \times 0 + 0.25 \times 0) = 0$$

$$\phi_d = 0$$

$$f = 1.05^{-1} (0.75 \times 19.52 + 0.25 \times 0) = 13.94$$

$$\phi = \frac{19.52 - 0}{110 - 90} = 0.976 \approx 0.98$$

Therefore, the price should be 13.94 at time t_0 .

Strategy: At time t_0 , you should borrow 34.06 and buy 0.98 ^{stock} shares of S . If the price goes up, borrow 2.2 from bank and buy extra ^{of stock} 0.02 share. Otherwise, sell out all stocks at time t_1 and put all cash in the bank.

